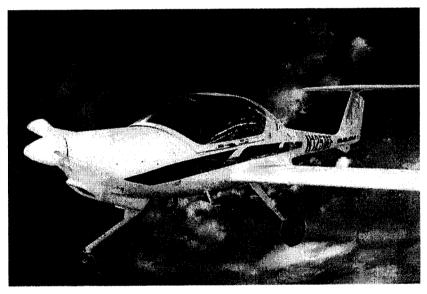
AE-324 Fundamentals of Atmospheric Flight Fall 2001

Research Project Report Team #6





December 7, 2001 Date Submitted:

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Note:

All the equations without specifying the reference are from, Anderson, John D., Jr. Introduction to Flight, Fourth Edition. McGraw-Hill, 2000

Information of the Aircraft (by Monal, Kosuke)

General Information:

Name of the Aircraft: Diamond Aircraft Katana 100

Price:

Propeller:

\$85,000 (Basic Model)

Manufacturer:

Diamond Aircraft Industries 1560 Crumlin Sideroad,

London, Ontario, Canada N5V 1S2

Phone: (519) 457-4000 Fax: (519) 457-4021

Specifications:

Hoffmann HO-V352F 2-blade, const. speed

Power Plant: Bombardier Rotax 912S, 100 hp max

Length: 23ft 6in Height: 6ft 11in

Wings: Airfoil: Wortmann FX 63-137/20 HOAC

Wingspan: 35ft 7in
Wing area: 125.0ft²
Aspect ratio: 10.0
Mean aerodynamic chord: 42.9in.

Dihedral: 4°
Sweep of leading edge: 1°

Height from ground: 3.05ft (Average)

Ailerons: Area: 7.08ft²
Wing flaps: Area 13.30ft²

Horiz. tail: Area: 18.21ft² 4.75ft²(elevator area)

Incident angle: -2.5°

Vertical tail: Area: 12.21ft² 4.59ft²(rudder area)

Weight: Empty weight: 1,095 lb

Maximum takeoff weight: 1,653 lb

Maximum landing weight: 1,653 lb

Wing Loading: 13.2 lbf/ft²
Power Loading: 16.5 lbf/hp

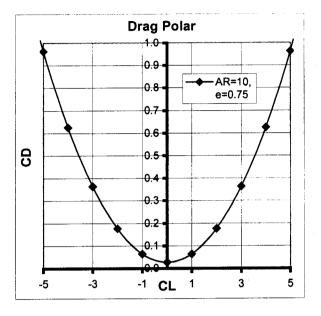
Fuel Capacity 20.1 gal (19.5 gal useable)

Aircraft Performance:

The aircraft Drag polar (by Kosuke)

The drag polar of the aircraft is determined as the right figure from the reasoning below.

The Profile drag C_{D,0} was derived from the catalog value of the level flight at 7,500ft with 75% power setting. The aircraft flies at 123 KTAS (207ft/s), since TAS stands for True Air Speed, the value is believed to be accurate. The Rotax 912S engine has the maximum power output of 100hp, so the power setting of 75% should produce 75hp at the sea level. To interpolate the power available at the 7,500ft, use



$$P_{A,alt} = \left(\frac{\rho_{alt}}{\rho_{SL}}\right) P_E \eta \left(\frac{\%}{100}\right) = \left(\frac{0.0018975}{0.0023769}\right) (55000) (0.85) \left(\frac{75}{100}\right) = 27990$$
 (EQN 1.1)

Where,

P_{A.alt} Power available at an altitude (lbf-ft/s)

 P_E Engine power at sea level (100hp = 55,000lbf-ft/s)

 ρ_{alt} Density of air at an altitude (0.0018975slugs/ft³ @7,500ft)

P_{SL} Density of air at sea level (0.0023769slugs/ft³)

η Propeller efficiency (0.85, see Appendix C)

% Power setting in %, between $0 \sim 100\%$

In a non-accelerated level flight, these two statements are true.

$$\begin{array}{lll} T = D, & also = \frac{1}{2}\rho_{\infty}V_{\infty}SC_D, & or = P \,/\, V_{\infty} & (EQN1.2a,b,c) \\ L = W, & also = \frac{1}{2}\rho_{\infty}V_{\infty}SC_L & (EQN1.3a,b) \end{array}$$
 Where,
$$\begin{array}{lll} T & Thrust \,(lbf); & D & Drag \,(lbf) \\ L & Lift \,(lbf); & W & Weight \,(lbf) \\ \rho_{\infty} & Density \,(slugs/ft^3); & V_{\infty} & Airspeed \,(ft/s) \\ S & Area \, of \, the \, wings \,(125 \, ft^2); & P & Power \, Required/Available \\ C_D & Coefficient \, of \, Drag; & C_L & Coefficient \, of \, Lift \end{array}$$

Using the equation 1.2c, the drag force on the airplane can be calculated as,

$$D = P_A / V_{\infty} = 134.82 \text{ lbf}$$

The coefficient of lift is determined by using the equation 1.3a, and 1.3b, (W=1,653 lbf).

$$C_L = \frac{2L}{\rho_n V_n^2 S} = \frac{2W}{\rho_n V_n^2 S}$$
 (EQN 1.4)

From the calculation, the $C_L = 0.3233$

Observing the equation 1.2 and 1.3, the ratio D/W is equal to C_D/C_L , The coefficient of drag (C_D) can be found from this relationship.

$$C_D = TC_L / W = (134.823)(0.3233)/(1653) = 0.026$$
 (EQN 1.5)

According to Raymer (See Appendix A), the Oswald efficiency of straight wing airplane can be approximated by using the formula described below.

$$e = 1.78(1 - 0.045AR^{0.68}) - 0.64$$
 (EQN 1.6)

Since the aspect ratio (AR) of Katana 100 is 10.0, so the Oswald efficiency is 0.756.

The profile of drag (C_{D,0}) is derived from the equation below,

$$C_D = \left(C_{D,0} + \frac{C_L^2}{\pi e A R}\right) = \left(C_{D,0} + \frac{(0.3233)^2}{\pi (0.756)(10)}\right)$$
 (EQN 1.7)

Bt solving this equation, $C_{D,0} = 0.022$.

Takeoff performance (by Kosuke)

The takeoff distance (SLO, ft) can be determined by using a formula,

$$S_{LO} = \frac{1.44W^2}{g\rho_{\infty}SC_{L,\text{max}} \left\{ T - \left[D + \mu_r(W - L)\right]_{0.7V_{LO}} \right\}}$$
 (EQN 2.1)

Where,

g Acceleration of Gravity (32.2ft/s²)

C_{L,max} Maximum coefficient of lift (refer equation)

 μr Rolling friction coefficient (paved runway =0.2)

The takeoff speed, V_{LO}, is determined to be 1.2V_{stall}, since Katana 100 stalls at 69.2ft/s,

$$V_{LO} = 1.2 (69.2) \text{ ft/s} = 83.04 \text{ ft/s} \text{ (with no head wind)}$$
 (EQN 2.2a)
Or, $V_{LO} = 1.2(69.2) - \text{(head wind in ft/s)}$ (EQN 2.2b)
= 68.37 ft/s (10mph head wind), 53.70 ft/s (20mph head wind)

As result, $0.7V_{LO}$, the value used to determine the average thrust, drag and lift during the takeoff become,

$$V_{avg} = 0.7V_{LO}$$
 (EQN 2.3)
= (0.7)(83.04) ft/s = 58.13 ft/s
also, = 47.86 ft/s, and 37.59 ft/s for 10mph, 20mph head wind

To find $C_{L,max}$, taking the stall speed as reference. The airspeed can be determined from the formula below,

$$V_{stall} = \sqrt{\frac{2W}{\rho_{\infty} SC_{L,\text{max}}}}$$
 (EQN 2.4)

by using this formula, the $C_{L,max}$ for 1,095lbf, 1,375lbf and 1,653lbf are to be 1.5392, 1.9328 and 2.3235 respectively.

From this condition, the average values of T, D and L for 1,095, 1,375 and 1,653lbf are,

T =	4940.95lbf,	4940.95lbf,	4940.95lbf	(refer EQN1.2c)
D =	42.74lbf,	59.58lbf,	80.07lbf	(refer EQN1.2b)
I. =	772.6lbf.	970.2lbf.	1166lbf	(refer EQN1.3b)

Also, the average values of T, D and L for 0, 10 and 20mph head wind are,

T =	4940.95lbf,	4068.09lbf	3195.22lbf	(refer EQN1.2c)
D =	59.58lbf,	40.39lbf,	20.92lbf	(refer EQN1.2b)
$I_{\cdot} =$	970.2lbf.	657.69lbf.	405.73lbf	(refer EQN1.3b)

- 1. Takeoff distance at different weight configurations (Sea level, no wind)
 - a. **781ft**, at empty weight (1,095lbf)
 - b. **987ft**, at medium weight (1,374lbf)
 - c. 1,196ft, at maximum gross take-off weight (1,653lbf)
- 2. Takeoff distance at different wind configurations (Sea level, medium weight)
 - a. 987ft, with 0mph head wind
 - b. 827ft, with 10mph head wind
 - c. 666ft, with 20mph head wind

Hodograph (by Monal)

The hodograph take vertical speed (or R/C) against horizontal speed. Both horizontal and vertical components of the velocity will be determined from the procedure described here.

- 1. Use the equation 1.1 to determine the PA
- 2. Drag force can be derived from equation 1.2a,b and c for desired speed.
- 3. Use equation 1.5, so the P_R will be determined
- 4. By using equation 3.1 shown below, the vertical component (V_v) of the air speed can be found.

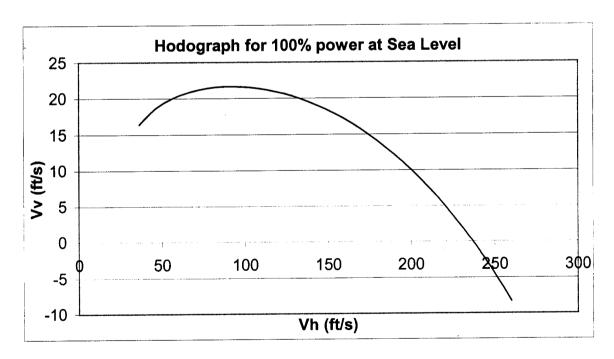
$$(R/C) = \frac{P_A - P_R}{W}$$
 (EQN 3.1)

5. Using the equation below, the climb angle of the airplane at the air speed will be found

$$\theta = \sin^{-1} \left(\frac{R/C}{V_m} \right)$$
 (EQN 3.2)

6. To find the horizontal component (V_h) , utilize the angle found from the equation 3.2.

$$V_h = V_\infty \cos \theta \tag{EQN 3.3}$$



Maximum Endurance and Maximum Range (by Kosuke)

According to the brochure, cruise condition is set at 7,500ft with 75% power setting. Katana 100 uses 6.1gph of fuel at cruise, and 1 gallon of the fuel weigh 5.64 lbf, thus the airplane consumes 34.40lbf of fuel per hour while the power available is 27,990lbf-ft/s (EQN1.1). The consistent unit (c) is,

$$c = 34.04lbf / [27990(lbf-ft/s)*3600(sec)] = 3.4139x10^{-7}(1/ft)$$
 (EQN 4.1)

By using Breguet's formulas, the maximum endurance and the range can be found from the two equations below. These equations are fairly accurate and have errors of 10~20%

$$E = \frac{\eta}{c} \left(\frac{C_L^{3/2}}{C_D} \right)_{\text{max}} \sqrt{2\rho_{\infty} S} \left(\frac{1}{\sqrt{W_1}} - \frac{1}{\sqrt{W_0}} \right)$$
 (EQN 4.2)

$$R = \frac{\eta}{c} \left(\frac{C_L}{C_D} \right)_{\text{max}} \ln \frac{W_0}{W_1}$$
 (EQN 4.3)

Where,

Range (ft) Endurance (sec.); R E

Density (slugs/ft³); Wing area (125 ft²) S ρ_{∞}

Weight at T=0 Propeller Efficiency (0.85); W₀

Weight after all usable fuel is consumed

In order to find the $(C_L/C_D)_{max}$ and $(C_L^{3/2}/C_D)_{max}$, these equations are used.

$$\left(\frac{C_L}{C_D}\right)_{\text{max}} = \frac{\left(C_{D,0}\pi eAR\right)^{1/2}}{2C_{D,0}} = 16.4283$$

$$\left(\frac{C_L^{3/2}}{C_D}\right)_{\text{max}} = \frac{\left(3C_{D,0}\pi eAR\right)^{3/4}}{4C_{D,0}} = 15.9194$$
(EQN 4.4)

$$\left(\frac{C_L^{3/2}}{C_D}\right)_{\text{max}} = \frac{\left(3C_{D,0}\pi eAR\right)^{3/4}}{4C_{D,0}} = 15.9194$$
(EQN 4.5)

Using the equation 4.2 and 4.3,

1. Maximum Endurance = 23,522 sec. = 6.53 hours

= 2,816,749 ft = 533 miles2. Maximum Range

Absolute and Service Ceilings (by Kosuke)

The service ceiling is defined as an altitude where $(R/C)_{max} = 100$ fpm. According to the catalog information, the service ceiling of Katana 100 is 17,600 ft.

The absolute ceiling is an altitude where the $(R/C)_{max} = 0$ fpm, so

$$(R/C) = \frac{P_A - P_R}{W} = 0$$
 (EQN 5.1a)

Where,

 P_A = Power available (lbf-ft/s)

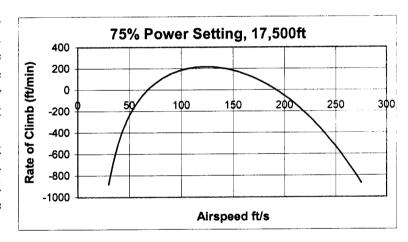
 P_R = Power required (lbf-ft/s)

W = weight of the airplane (1,653 lbf)

The performance at the service ceiling can be used to determine the absolute ceiling of the airplane. Using equation 5.1a the difference between P_A and P_R is,

$$P_A-P_R = (100 \text{ft/min})(1 \text{min/60sec.})(1,653 \text{lbf}) = 2,755 \text{ lbf-ft/s}$$
 (EQN 5.1b)

The mathematical model at 17,500ft has the rate of climb of more than 200fpm (shown in right). In order to interpolate the value, a new P_A value must to be selected. The curve of the graph may move vertically as the P_A changes, but the related air speed remains the same. In this case, the V_{∞} , which gives the best R/C, is at 124ft/s. By using equation 1.4 and 1.7, the C_L is equal to 1.2481, and the drag is 169.22lbf. The P_R can be determined by referring to equation 1.2c,



$$P_R = DV_{\infty} = (169.22lbf)(124ft/s) = 13543 lbf-ft/s$$

 $P_A = P_R + 2,755lbf-ft/s = 13,543+2,755 = 16,928 lbf-ft/s$

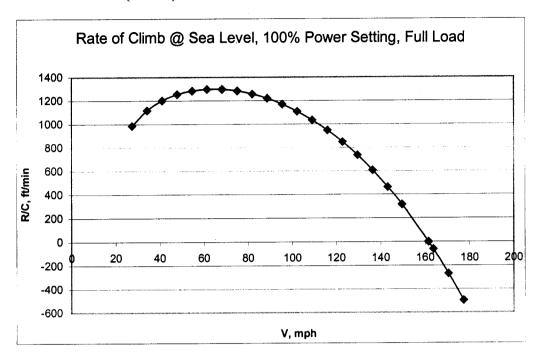
From EQN1.1,
$$P_A = (P_E)\eta(\rho_{alt}/\rho_{sea\ level}) = (P_E)(0.85)(0.0013781/0.0023769)$$

 $P_E = 33,054lbf-ft/s = 60\ hp$ (equivalent power output at sea level)

Recalculating the rate of climbs by every 500ft with $P_E = 60hp$, the absolute ceiling is about 19,500ft

Climbing Performance (by Monal, Kosuke)

1. Rate of Climb (Monal)



In order to a create rate of climb graph, C_L, C_D, D, T_R, P_R, P_A must be calculated for airspeed. The equations used are,

- Equation 1.4 to determine C_L
- Equation 1.3b and 1.7 for C_D and drag force (D)
- Equation 1.5 for the thrust required (T_R)
- Equation 1.2 for the power required (P_R)
- Equation 1.1 for the power available (P_A)
- Equation 5.1 to determine the Rate of Climb

By using the equations above, rate of climb charts for any desired altitude can be derived.

2. Time to Climb (Kosuke)

Katana 100 has a Rotax 912S engine, which allows 100% power for 5 minutes, and/or 75% continuous power. Equation 5.1a is used to calculate the time to climb for every 500ft. As the result,

- a. To 5,000ft
 b. To 10,000ft
 c. To 15,000ft
 386 seconds / 255 seconds (w/ 5 min. 100% operation)
 yof seconds / 760 seconds (w/ 5 min. 100% operation)
 1,673 seconds / 1,527 seconds (w/ 5 min. 100% operation)
- Time to Climb 16000 14000 12000 Attitude (#) 10000 8000 6000 4000 4000 75% 2000 = 5min 100%, -> 75% 0 1600 1800 1000 1200 1400 600 800 200 400 0 Time (second)

Maximum level flight speed (by Monal)

From observing the rate of climb chart shown in the previous section, the maximum level flight occurs at where the x-axis and the graph curve intersects. By drowning the charts for sea level, 5,000ft and 10,000ft, the values for each case are determined.

1. Sea level 235ft/s (161mph)
2. 5,000ft 234ft/s (160mph)
3. 10,000ft 231ft/s (157mph)

V-n Diagram (by Monal)

For general aviation, the limit load factors are 6 at positive, and -3 at negative side for aerobatic category. This means the airplane has to stand the both loading conditions. Since the C_L value for the negative side is not available, the value was assumed to be the same as for the positive side.

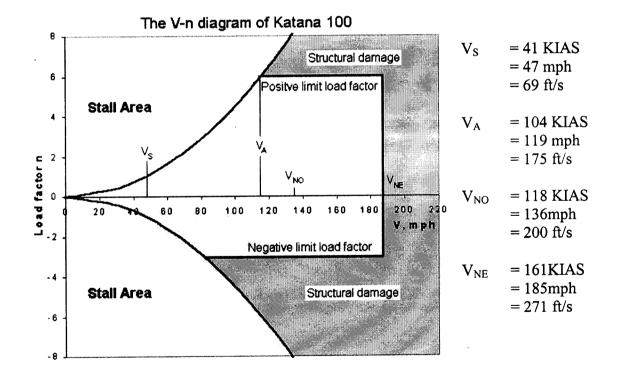
To determine the load factor (n),

$$n = \frac{L}{W} = \frac{\frac{1}{2} \rho_{\infty} V_{\infty}^2 SC_L}{W}$$
 (EQN 8.1)

From the specification, the wing loading is $13.2 \, lbf/ft^2$ which is weight of the aircraft over the wing area (=W/S). And for the maximum load factor for any airspeed, the C_L must be set at the maximum value, which is 2.3253, referred from equation 2.4, at the takeoff performance section. For n_{max} , the equation $8.1 \, becomes$,

$$n_{\text{max}} = \frac{1}{2} \rho_{\infty} V_{\infty}^2 \frac{C_{L,\text{max}}}{W/S}$$
 (EQN 8.2)

By using this equation, the V-n diagram has been created as shown below. (The catalog values shown as references)



Appendix: (by Kosuke)

Appendix A: References

Anderson, John D., Jr. Introduction to Flight, Fourth Edition. McGraw-Hill, 2000

Raymer, Daniel P. Aircraft Design: A Conceptual Approach, Third Edition. AIAA, 1999

Diamond Aircraft Industries Hoffmann Propeller Germany

http://www.hoffmann-prop.com/

http://www.diamondair.com/

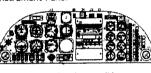
Zenith Aircraft Company NASG Airfoil Database

http://www.zenithair.com/kit-data/zac-rtx912s.html http://www.nasg.com/afdb/list-polar-e.phtml

Appendix B: Aircraft Brochure

Katana DA20-A1

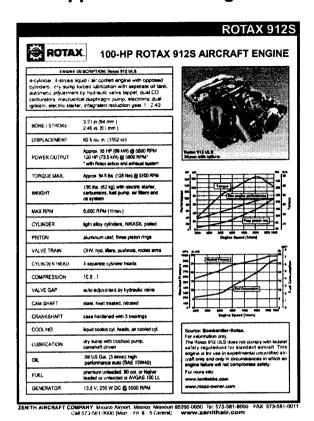
Katana DA20-A1







Appendix C: Engine Performance Sheet



Typical Power, Torque and Fuel Consumption For Rotax 4 Stroke Aircraft Engines

		912 UL		912 ULS				UL	
Power Setting (%)	85	75	1001	65	75. 1	100*	65	75	100
RPM	4800	5000	5800	4800	5000	5000	4800	5000	5800
Power									
Kilowatta	37.7	43.5	59.8	44.6	51.0	73.5	47.8	55.1	84.6
Horsepower	50.6	58.4	81.0	59.8	68.4	100.0	64.1	73.9	115.
Torque									
Nin	75.0	83.0	98.1	88.7	97.4	121.0	95.1	105.2	130
Ft.*IDs.	55.3	61.2	72.3	85.4	71.8	89.2	70.1	77.6	102.
Fuel Consumption									
Litres / Hour	15.8	19,2	23.8	18.0	20.0	27.0	16.3	21.4	34 3
U.S. Gal. / Hour	4.2	5.1	6.3	4.6	5.3	7.1	4.8	5.7	9.6
Imp. Gal. / Hour	3.5	4.2	5.2	4.0	4.4	5.9	4.0	4.7	7.

^{*} Muximum take off power

Specifications and performance data shown is typical for the engines described and is general in nature All information is subject to change without notice. Information atnown is not to be used for flight or flight plunning purposes. Consult the appropriate engine endfor nazoral manualists for expedict technical information.

Appendix D: Propeller Performance

