

12<sup>th</sup>, Oct, 2002

# MathCAD Quick Start Guide

### Crosshair Cursor :

By clicking anywhere on a blank portion of the document, a red crosshair cursor will appear. If you proceed to type, MathCAD will enter into either formula editing mode, or text mode. Default mode is "Formula editing mode". In this mode, you can perform **Assignment of Constants and functions**. To enter **Text mode**, type some letter and then hit the space bar.

### Text mode: ( " )

In MathCAD, you can type as well as formulas. There are two ways to enter into text model.

- 1) By typing ( " ) .
- 2) After typing some letters, then press space bar.

### Editing a Text Box :

- 1) Activate the text box.
  - a) by clicking inside the text area.
  - b) Clicking and dragging a box around the text area.
- 2) Move the cursor with the mouse or arrow keys to navigate to the place that you want to edit.
- 3) CTRL + ARROW : skips over words.  
CTRL + SHIFT + ARROW : highlights words.

Sub/super script : **Select** the terms that you want to change, then **right click**, and select **FONT**. then you can find the box called "Effects".

Box / Highlight : **Select** the terms that you want to change, then **right click**, and select **Properties**.

Then put the **check** for Highlight or show the border.

Greek Letter : In "Formula editing mode", type in a letter, then "CTRL + g".

### Moving a Text Box :

- 1) Activate the text box.
- 2) Move the cursor to the boundary of the text area or formula until the cursor turns into a hand.
- 3) Drag and drop at wherever you want.

### Resizing the Text Box :

- 1) Activate the text box.
- 2) Click on the lower right black square in the text box.
- 3) Drag that square to resize the text box.

### Constants :

#### Assignment of Constants : (:)

Example: assignment of the value "86" to the variable "a" :

a := 86                    [ a : 86 <Enter> ]

#### Displaying Constan's Value :

There are two ways to display the values, numerically and symbolically.

Example: reading the value of a constant "a" :

a = 86                    ( Calculating )    [ a = <Enter> ]

a → 86                    ( Evaluating )    [ a CTRL+ . <Enter> ]

Symbolic calculation is very powerfull and useful, however, it requires lots of CPU power. Simply it takes much longer to get the solution.

## Functions :

Functions are used when you want an expression to accept some parameter, perform some mathematical operation, and then output the result. These functions may be then later included in other functions.

### Assignment of Functions :

Example : assigning the function "f" of "x" :

#### **notes :**

$$f(x) := 2 \cdot x - a$$

[ f ( x ) : 2 \* x - a <Enter>]

- 1) Everything which is not predefined must be in the parameter list.  
The parameter list is the list of variables between the parentheses.
- 2) The constant "a" is already defined above, so the value of "a" is automatically substituted into this expression.
- 3) The variable "x" is not defined above, then it must be defined in the parameter list.
- 4) Some of the major special constants are already defined, such as  $\pi$ .

Example : assigning the function "g" as a function of "f" :

$$g(x) := a \sin(f(x)) - a \quad [ g ( x ) : a * \sin ( f ( x ) ) + 4 <Enter>]$$

**notes :** If there is a constant which is not defined above, MathCAD will give you an error message.

$$h(x) := 3 \cdot x^2 + b \cdot x$$

### Displaying Functions' Value :

Example: finding the value of a function "f" when "x=2" :

$$f(2) = -82 \quad [ f ( 2 ) = ]$$

$$g(2) = -112.938 \quad [ g ( 2 ) = ]$$

$$h(2) = \blacksquare \quad [ h ( 2 ) = ]$$

Since constant "b" is not defined above, MathCAD cannot find the value. However, by using an "Evaluation" instead of using a "Calculation",

$$h(2) \rightarrow 12 + 2 \cdot b \quad [ h ( 2 ) \text{ Ctrl+. } ]$$

If you want to see the function itself, then...

$$h(x) \rightarrow 3 \cdot x^2 + b \cdot x \quad [ h ( x ) \text{ Ctrl+. } ]$$

$$\frac{d}{dx} h(x) \rightarrow 6 \cdot x + b \quad [ \text{Shift+ / } h ( x ) \text{ Ctrl+. } ]$$

Also, by using an "Evaluation" function, you can do....

$$h(x) \text{ factor} \rightarrow x \cdot (3 \cdot x + b) \quad [ h ( x ) \text{ Ctrl+Shift . factor } ]$$

$$h(x) \text{ substitute, } b = 5 \rightarrow 3 \cdot x^2 + 5 \cdot x \quad [ h ( x ) \text{ Ctrl+Shift . substitute , } b \text{ Ctrl+= 5 } ]$$

$$g(2) \text{ float, } 5 \rightarrow -112.94 \quad [ g ( 2 ) \text{ Ctrl+Shift . float , } 5 ]$$

Other useful command for the "Evaluation" functions are

- simplify, expand (same manner as "factor")
- collect (same manner as "float")

Moreover, you can use a combination like.....

$$h(x) \left| \begin{array}{l} \text{substitute, } b = 5 \\ \text{factor} \end{array} \right. \rightarrow x \cdot (3 \cdot x + 5) \quad [ h ( x ) \text{ Ctrl+Shift . substitute , } b \text{ Ctrl+= 5} \\ <\text{space}><\text{Space}> \text{ Ctrl+Shift . factor } ]$$

### Expression Navigation :

Expression navigation may be somewhat tricky at the first time, but you can easily get use to it.

#### **Key:**

- 1) Always look at the blue underline.
- 2) The next mathematical operation will be performed on the group which the blue underline is covering.
- 3) By pressing the space bar, you can enlarge the range of your blue underline.

$$y(x) := 2 \cdot x^2 \quad [ y ( x ) : 2 * x ^ 2 ]$$

$$y(x) := 2 \cdot x^{\frac{2}{3}} \quad [ y ( x ) : 2 * x ^ 2 / 3 ]$$

$$y(x) := 2 \cdot \frac{x^2}{3} \quad [ y ( x ) : 2 * x ^ 2 <Space> / 3 ]$$

$$y(x) := \frac{2 \cdot x^2}{3} \quad [ y ( x ) : 2 * x ^ 2 <Space><Space> / 3 ]$$

### Calculus Functions :

**Differentiation and Integration :** < Shift + / > and < Ctrl + i >

Example: Find the derivative of "f" :

< recall the previous function >

$$f(x) \rightarrow 2 \cdot x - 86 \quad [ f ( x ) \text{ Ctrl+ . } ]$$

< Define the function for a differentiation and Integration >

$$f_d(x) := \frac{d}{dx} f(x) \quad [ f . d ( x ) : \text{ Shift + / } f ( x ) <tab> x ]$$

$$f_I(x) := \int f(x) dx \quad [ f . I ( x ) : \text{ Ctrl + i } f ( x ) <tab> x ]$$

< Now display those functions >

$$f_d(x) \rightarrow 2 \quad [ f . d ( x ) \text{ Ctrl+ . } ]$$

$$f_I(x) \rightarrow x^2 - 86 \cdot x \quad [ f . I ( x ) \text{ Ctrl+ . } ]$$

#### **notes :**

Sometimes the calculus functions can be tricky. Even if your function is looked correct to you, sometimes MathCAD tells you a wrong answer.

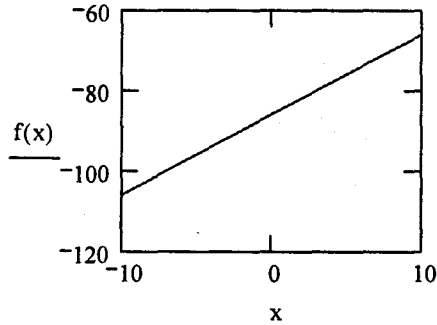
In this case, you can try two things,

- 1) retype your equation
- 2) try to put parenthesis around a complicate expression.

**Graphing :**

**Graphing Functions :**

Example: Graphing the function "f" : [ Shift + 2 <tab> f(x) ]

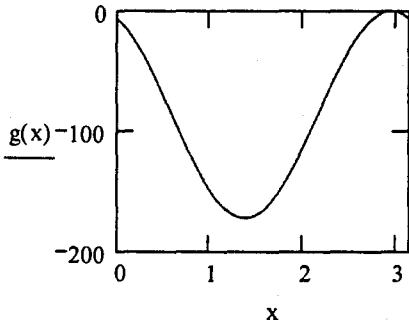


**notes:**

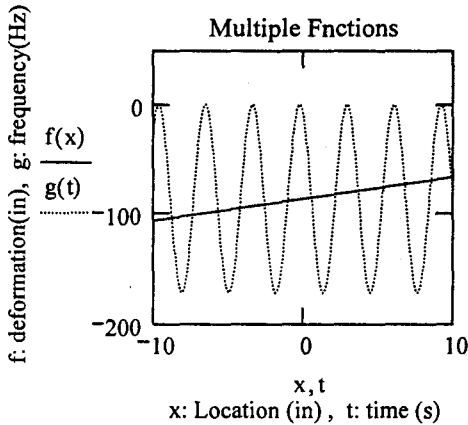
- 1) By pressing <tab> or <arrow>, you can skip the domain variable.
- 2) When you do not define the domain, MathCAD automatically sets the domain to be " [10, 10] "

Example: Defining a Domain while Graphing :

[ Shift+2 x Shift+<tab> 0 <tab> Ctrl+Shift+p <tab> g(x) ]

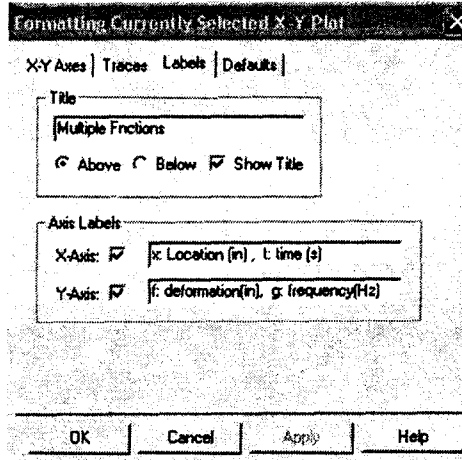


Example: Graphing Multiple functions, "f", and "g" : [ Shift + 2 <tab> f(x) , g(x) ]



**notes:**

- 1) You can manually change the ranges of the graph and variables for each function anytime.
- 2) You can resize the graph in the same way as a text box by clicking and dragging.
- 3) You can put the title for your plot and change the type of plot lines by double-clicking the plots.



**For the case of Piece-wise Functions :**

< Show all the function we have now >

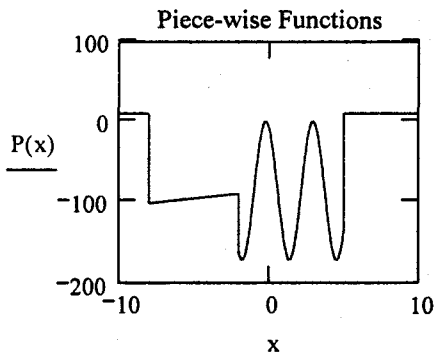
$$f(x) \rightarrow 2 \cdot x - 86$$

$$g(x) \rightarrow 86 \cdot \sin(2 \cdot x - 86) - 86$$

< Now we define the piece-wise Function by using "Add line" >

$$P(x) := \begin{cases} f(x) & \text{if } -8 < x < -2 \\ g(x) & \text{if } -2 \leq x < 5 \\ 10 & \text{otherwise} \end{cases}$$

[ p(x) : ] up-arrow up-arrow  
 f(x) } -8<x<-2 <tab>  
 g(x) } -2 Ctrl+9 x < 5 <tab>  
 y(x) Ctrl+Shift+] ]



[ Shift+2 x <tab> P(x) <Enter>  
 Double click, click the tab for "labels" ]

**Vector : (1D, 2D and multidimensional Arrays)**

**Basic Array Assignments :**

example: Assigning by creation of a vector and then displaying the contents :

$$m := \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} \quad [ m : \text{Ctrl}+m \ 3 \ \text{<tab> } 1 \ \text{<Enter> } 1 \ \text{<tab> } 2 \ \text{<tab> } 3 ]$$

$$m = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} \quad [ m = ] \quad m^T = (1 \ 2 \ 3) \quad [ m \ \text{Ctrl}+1 = ]$$

**note :** It is very nice to use transpose function to save the space.

$$n := (4 \ 5 \ 6)^T \quad [ n : \text{Ctrl}+m \ 1 \ \text{<tab> } 3 \ \text{<Enter> } 4 \ \text{<tab> } 5 \ \text{<tab> } 6 \\ \text{<right arrow> Ctrl+1}]$$

$$n^T = (4 \ 5 \ 6) \quad [ n \ \text{Ctrl}+1 = ]$$

example : displaying each components of the vectors

$$n_0 = 4 \quad [ n [ 0 = ]$$

$$n_1 = 5 \quad [ n [ 1 = ]$$

example : assigning by referencing indices

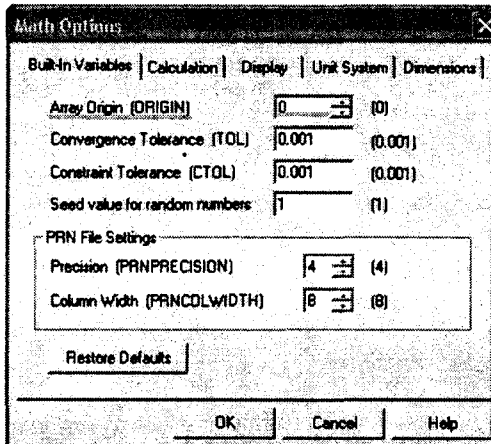
$$n_4 := 8 \quad [ n [ 3 : 8 ]$$

$$n^T = (4 \ 5 \ 6 \ 0 \ 8) \quad [ n \ \text{Ctrl}+1 = ]$$

**note :** Since 3rd row was skipped, MathCAD automatically takes it "0"(zero).

**note :**

The base index of the vector is zero. It is possible to change the starting index by going into **options** under the **math** menu. [ alt+m O ]



**Subscript and Index:**

- 1) m subscript 1 is defined for the constant value.
- 2) m index 1 is defined for the value under the vector named "m".

$$m_1 := 3 \quad [m . 1 : 3]$$

$$m_1 = 3 \quad [m . 1 = ]$$

$$m_1 = 2 \quad [m [ 1 = ]$$

It is possible to combin subscripting with indexing :

$$m_{\alpha_1} := 1 \quad [m . a \text{ Ctrl+g } [ 1 : 1 ]$$

$$m_{\alpha_2} := 2 \quad [m . a \text{ Ctrl+g } [ 2 : 2 ] \quad m_{\alpha}^T = (0 \ 1 \ 2) \quad [m . a \text{ Ctrl+1 = ]$$

**Iterative Vector Assignments & Range Constants:**

**example:** Basic Iterative Assignment

$$m := 0 \quad [m : 0] \quad < \text{Clearing up old vector data} >$$

$$i := 1..5 \quad [i : 1;5] \quad < \text{Assigning a range for i from 1 to 5} >$$

**note :** normally increment of range constant is "1" (one).

If you want to use other increment, (for example : 0.2 ), you need to type....

$$j := 1,1.2..2 \quad [j : 1, 1.2 ; 2]$$

$$m_i := 5 \cdot i \quad [m [i : 5 * i] \quad \text{note: MathCAD automatically does a for-loop over the index}$$

$$m^T = (0 \ 5 \ 10 \ 15 \ 20 \ 25) \quad [m \text{ Ctrl+1 = ]$$

**note:** Since the default starting index is "0" (zero), the vector , m , is started with 0<sup>th</sup> index. then totally it has 6 (six) rows.

j =
1
1.2
1.4
1.6
1.8
2

**example :** index constant with modifications

$$m := 0 \quad [m : 0] \quad < \text{Clearing up old vector data} >$$

$$m_{2+i} := 2 \cdot i \quad [m [i : 5 * i] \quad \text{note: the expression in the index must evaluate to an integer value!}$$

$$m^T = (0 \ 0 \ 0 \ 2 \ 4 \ 6 \ 8 \ 10) \quad [m \text{ Ctrl+1 = ]$$

**example : 2-Dimensional Arrays**

$p := 1..9$        $[p : 1 ; 9]$

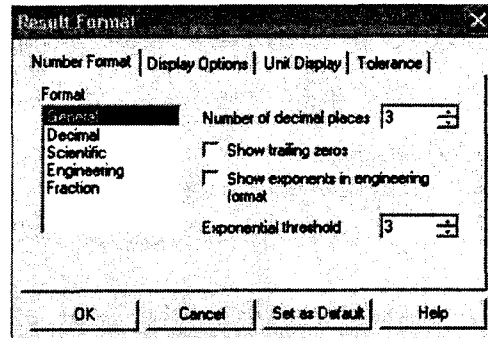
$q := 1..9$        $[q : 1 ; 9]$

$N_{p,q} := p \cdot q$        $[N [ p , q <space><space> : p * q ]$

	0	1	2	3	4	5	6	7	8	9
0	0	0	0	0	0	0	0	0	0	0
1	0	1	2	3	4	5	6	7	8	9
2	0	2	4	6	8	10	12	14	16	18
3	0	3	6	9	12	15	18	21	24	27
4	0	4	8	12	16	20	24	28	32	36
5	0	5	10	15	20	25	30	35	40	45
6	0	6	12	18	24	30	36	42	48	54
7	0	7	14	21	28	35	42	49	56	63
8	0	8	16	24	32	40	48	56	64	72
9	0	9	18	27	36	45	54	63	72	81

**note :**

It is possible to change the result format. By double clicking your result, the window shown below will pop up. You can change, decimal place, format of your matrix and etc....



**example : 3-Dimensional Arrays ( Set the array origin 1 (one).)**

$p := 1..3$      $q := 1..3$      $r := 1..3$        $[p : 1 ; 3]$     $[q : 1 ; 3]$     $[r : 1 ; 3]$

$A_{p,q} := p \cdot q$      $A = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 2 & 3 \\ 0 & 2 & 4 & 6 \\ 0 & 3 & 6 & 9 \end{pmatrix}$

$[A [ p , q <space><space> : p * q ]$        $[A =]$

$B_r := r \cdot A$

$[B [ r <space> : r * A ]$

$[B =]$   
 $B = \begin{pmatrix} 0 \\ \{4,4\} \\ \{4,4\} \\ \{4,4\} \end{pmatrix}$

$[B [1 =]$   
 $B_1 = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 2 & 3 \\ 0 & 2 & 4 & 6 \\ 0 & 3 & 6 & 9 \end{pmatrix}$

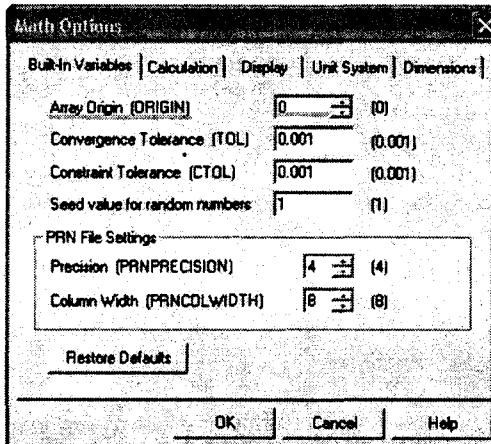
$[B [2 =]$   
 $B_2 = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 2 & 4 & 6 \\ 0 & 4 & 8 & 12 \\ 0 & 6 & 12 & 18 \end{pmatrix}$

$[B [3 =]$   
 $B_3 = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 3 & 6 & 9 \\ 0 & 6 & 12 & 18 \\ 0 & 9 & 18 & 27 \end{pmatrix}$



**note :**

The base index of the vector is zero. It is possible to change the starting index by going into **options** under the **math** menu. [ alt+m O ]



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$$m_{\alpha}^T = (0 \ 1 \ 2) \quad [m . a \text{ Ctrl+1 = ]$$

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If you want to use other increment, (for example : 0.2 ), you need to type....

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$$m^T = (0 \ 5 \ 10 \ 15 \ 20 \ 25) \quad [m \text{ Ctrl+1 = ]$$

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1.4
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**example :** index constant with modifications

$$m := 0 \quad [m : 0] \quad < \text{Clearing up old vector data} >$$

$$m_{2+i} := 2 \cdot i \quad [m [i : 5 * i] \quad \text{note: the expression in the index must evaluate to an integer value!}$$

$$m^T = (0 \ 0 \ 0 \ 2 \ 4 \ 6 \ 8 \ 10) \quad [m \text{ Ctrl+1 = ]$$

**example : 2-Dimensional Arrays**

$p := 1..9$        $[p : 1 ; 9]$

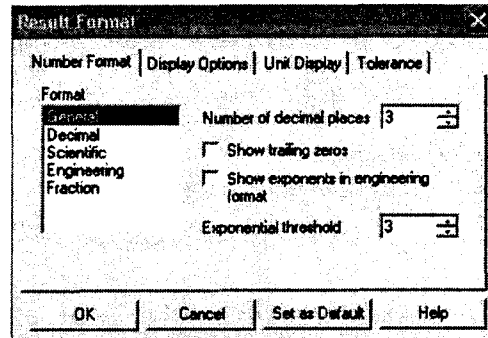
$q := 1..9$        $[q : 1 ; 9]$

$N_{p,q} := p \cdot q$        $[N [ p , q <space><space> : p * q ]$

	0	1	2	3	4	5	6	7	8	9
0	0	0	0	0	0	0	0	0	0	0
1	0	1	2	3	4	5	6	7	8	9
2	0	2	4	6	8	10	12	14	16	18
3	0	3	6	9	12	15	18	21	24	27
4	0	4	8	12	16	20	24	28	32	36
5	0	5	10	15	20	25	30	35	40	45
6	0	6	12	18	24	30	36	42	48	54
7	0	7	14	21	28	35	42	49	56	63
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**note :**

It is possible to change the result format. By double clicking your result, the window shown below will pop up. You can change, decimal place, format of your matrix and etc....



**example : 3-Dimensional Arrays ( Set the array origin 1 (one).)**

$p := 1..3$      $q := 1..3$      $r := 1..3$        $[p : 1 ; 3]$     $[q : 1 ; 3]$     $[r : 1 ; 3]$

$A_{p,q} := p \cdot q$      $A = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 2 & 3 \\ 0 & 2 & 4 & 6 \\ 0 & 3 & 6 & 9 \end{pmatrix}$

$[A [ p , q <space><space> : p * q ]$        $[A =]$

$B_r := r \cdot A$

$[B [ r <space> : r * A ]$

$[B =]$   
 $B = \begin{pmatrix} 0 \\ \{4,4\} \\ \{4,4\} \\ \{4,4\} \end{pmatrix}$

$[B [1 =]$   
 $B_1 = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 2 & 3 \\ 0 & 2 & 4 & 6 \\ 0 & 3 & 6 & 9 \end{pmatrix}$

$[B [2 =]$   
 $B_2 = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 2 & 4 & 6 \\ 0 & 4 & 8 & 12 \\ 0 & 6 & 12 & 18 \end{pmatrix}$

$[B [3 =]$   
 $B_3 = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 3 & 6 & 9 \\ 0 & 6 & 12 & 18 \\ 0 & 9 & 18 & 27 \end{pmatrix}$

### Vector Functions :

If you need to define the series of function from  $x^1$  to  $x^3$ , then it can be simply be done....

$$\begin{aligned} i &:= 1..3 && [i : 1 ; 3] \\ f(x)_i &:= x^i && [f(x) [i : x ^ i] \end{aligned}$$

However, this expression does not work. To get around this error, you can either explicitly declare the function such as,

$$\phi(x) := \begin{pmatrix} 1 & x & x^2 \end{pmatrix}^T \quad [f \text{ ctrl+g } (x) : \text{Ctrl+m } 1 \text{ <tab> } 3 \text{ <enter> } 1 \text{ <tab> } x \text{ <tab>} \\ &&& x \text{ shift+6 } 2 \text{ <space> <space> Ctrl+1 ]$$

Or you could declare a function using the programming tools, "Add line".

$$\phi(x) := \left| \begin{array}{l} \text{for } i \in 1..3 \\ a_i \leftarrow x^i \\ \phi \leftarrow a \end{array} \right.$$

### Solving Systems of Equations :

Example : Using a solve block

$$X + 6Y = 3 \qquad 3X - 5Y = 2$$

To solve this problem, you need to type in the initial values for x and y.

< Initial Guess >     $X := 1$      $Y := 1$

Given

**note** : The word " given" must be typed in equation mode.

$$\begin{aligned} X + 6Y = 3 & \quad [X + 6 * Y \text{ Ctrl+= } 3] \\ 3X - 5Y = 2 & \quad [3 * X - 5 * Y \text{ Ctrl+= } 2] \end{aligned}$$

$$\text{Find}(X, Y) = \begin{pmatrix} 1.174 \\ 0.304 \end{pmatrix}$$

It is possible to ignore "Initial Guess".

Given

**note** : The word " given" must be typed in equation mode.

$$\begin{aligned} S + 6T = 3 & \quad [S + 6 * T \text{ Ctrl+= } 3] \\ 3S - 5T = 2 & \quad [3 * S - 5 * T \text{ Ctrl+= } 2] \end{aligned}$$

$$G := \text{Find}(S, T) \qquad [G : \text{Find} ( S , T )]$$

$$G^T \rightarrow \begin{pmatrix} \frac{27}{23} & \frac{7}{23} \end{pmatrix} \qquad [G \text{ Ctrl+1}]$$

$$G_1 \text{ float, 4} \rightarrow 1.174 \qquad [G [ 1 \text{ Ctrl+Shift+. float , 4 <enter> } ]]$$

$$G_2 \text{ float, 4} \rightarrow .3043 \qquad [G [ 2 \text{ Ctrl+Shift+. float , 4 <enter> } ]]$$

Example : Using a matrix and row reduction to solve for a linear system of equations

$$\begin{pmatrix} 2 & 5 & 6 \\ 3 & 4 & 1 \\ 5 & 2 & 3 \end{pmatrix} \cdot \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} -3 \\ 2 \\ -1 \end{pmatrix}$$

First you need to create a augmented matrix with the coefficients on the left and the solution on the right.

" Row Reduced Echelon Form "

$$\text{rref} \left( \begin{pmatrix} 2 & 5 & 6 & -3 \\ 3 & 4 & 1 & 2 \\ 5 & 2 & 3 & -1 \end{pmatrix} \right) = \begin{pmatrix} 1 & 0 & 0 & 0.202 \\ 0 & 1 & 0 & 0.619 \\ 0 & 0 & 1 & -1.083 \end{pmatrix}$$

Thus the solution to the above system of equation is :

$$x = 0.202, y = 0.619, z = -1.083$$

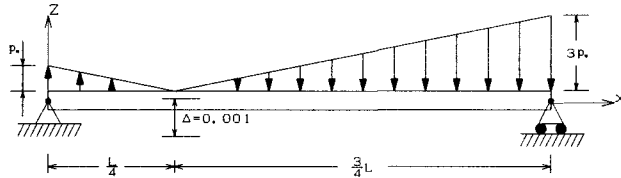
Example : Finding a roots of a polynomial with " root " .

$$S(x) := x^3 + 3x^2$$

$$\text{root}(S(x), x) \rightarrow (0 \ 0 \ -3)$$

1) A simply supported Euler-Bernoulli beam is subjected to a distributed force per unit length as shown below. In addition, the beam is subjected to a prescribed displacement,  $\Delta$ , at  $x=L/4$ . Determine the exact solution for the transverse deflection,  $w(x)_{\text{exact}}$ .

- $EI=20 \times 10^6 \text{ lb-in}^2$
- $L=20 \text{ in}$
- $P_0=40 \text{ lb/in}$
- $\Delta=0.001 \text{ in}$



**<Given values>**

**<Beam property >**    **<The size of the Entire Domain>**

$EI := 20 \cdot 10^6$      $L := 20$  (ft)

**<Define the Distributed Load and point force for Local elements>**

**(Magnitude of the Distributed Load)**    **(Distributed transverse Load)**

$P_0 := 40$  (lb)

$p_z(x) := P_0 - \frac{4 \cdot P_0}{L} \cdot x$  (lb/in)

**<Exact Solution>**

(GDE)  $EI v_4(x) := p_z(x)$      $EI v_4(x) \rightarrow 40 - 8x$

(Segment 1 :  $0 \leq x < \frac{L}{4}$  )

(Segment 2 :  $\frac{L}{4} \leq x \leq L$  )

$-Vy(x) : EI v_{13}(x) := 40x - 4x^2 + c_1$

$EI v_{23}(x) := 40x - 4x^2 + c_5$

$Mz(x) : EI v_{12}(x) := 20x^2 - \frac{4}{3}x^3 + c_1x + c_2$

$EI v_{22}(x) := 20x^2 - \frac{4}{3}x^3 + c_5x + c_6$

Slope :  $EI v_{11}(x) := \frac{20}{3}x^3 - \frac{1}{3}x^4 + \frac{1}{2}c_1x^2 + c_2x + c_3$

$EI v_{21}(x) := \frac{20}{3}x^3 - \frac{1}{3}x^4 + \frac{1}{2}c_5x^2 + c_6x + c_7$

Deflection :  $v_1(x) := \frac{1}{EI} \left( \frac{5}{3}x^4 - \frac{1}{15}x^5 + \frac{1}{6}c_1x^3 + \frac{1}{2}c_2x^2 + c_3x + c_4 \right)$      $v_2(x) := \frac{1}{EI} \left( \frac{5}{3}x^4 - \frac{1}{15}x^5 + \frac{1}{6}c_5x^3 + \frac{1}{2}c_6x^2 + c_7x + c_8 \right)$

**<Solve for the those constant values>**

Given **No deflection at the supports :**  
 $v_1(0) = 0$      $v_2(L) = 0$

**No Moment at the pin connections :**  
 $EI v_{12}(0) = 0$      $EI v_{22}(L) = 0$

**Deflection at L/4 is -0.001 (in) :**

**Compatibility for Moment at  $x=L/4$  :**

$v_1\left(\frac{L}{4}\right) = -0.001$      $v_2\left(\frac{L}{4}\right) = -0.001$

$EI v_{12}\left(\frac{L}{4}\right) = EI v_{22}\left(\frac{L}{4}\right)$

**Compatibility for the slope at L/4 :**

**Force equilibrium :**

$EI v_{11}\left(\frac{L}{4}\right) = EI v_{21}\left(\frac{L}{4}\right)$

$EI v_{13}(0) - EI v_{23}(L) + V_z + \int_0^L p_z(x) dx = 0$

**Constant Values :**

$C := \text{Find}(c_1, c_2, c_3, c_4, c_5, c_6, c_7, c_8, V_z) \text{ float}, 4 \rightarrow$

-140.0
0
-3583.
0
224.4
-1822.
972.2
-7593.
364.4

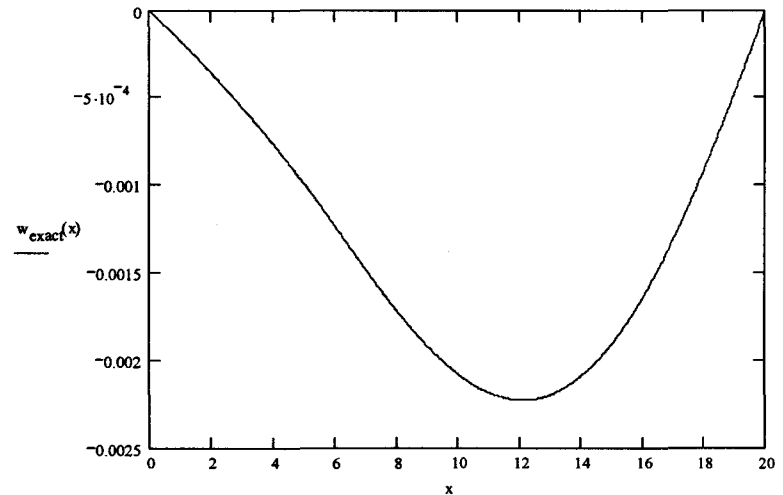
**<Re-define the function for the deflection curve>**

$C^T = (-140 \ 0 \ -3.583 \times 10^3 \ 0 \ 224.4 \ -1.822 \times 10^3 \ 972.2 \ -7.593 \times 10^3 \ 364.4)$

(Segment 1 :  $0 \leq x < \frac{L}{4}$  )     $v_1(x) := \frac{1}{EI} \left( \frac{5}{3}x^4 - \frac{1}{15}x^5 + \frac{1}{6}C_1x^3 + \frac{1}{2}C_2x^2 + C_3x + C_4 \right)$

(Segment 2 :  $\frac{L}{4} \leq x \leq L$  )     $v_2(x) := \frac{1}{EI} \left( \frac{5}{3}x^4 - \frac{1}{15}x^5 + \frac{1}{6}C_5x^3 + \frac{1}{2}C_6x^2 + C_7x + C_8 \right)$

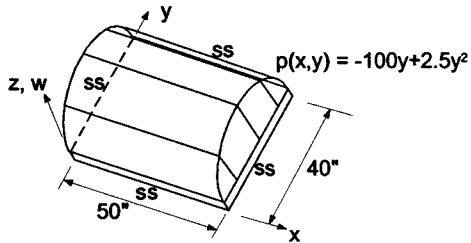
**Deflection curve over the domain :**  $w_{\text{exact}}(x) := \begin{cases} v_1(x) & \text{if } x \leq \frac{L}{4} \\ v_2(x) & \text{otherwise} \end{cases}$



2) Consider the rectangular plate below with all edges simply supported. The 0.2" thick plate is made from a steel alloy with a Young's modulus of  $3(10)^7$  psi and Poisson's ration of 0.3. The transverse load on all points of the upper surface,  $p(x, y)$ , is independent of  $x$  and can be expressed as

$$p = -100 \cdot y + 2.5 \cdot y^2$$

Find the Navier solution for the transverse deflections of the plate at (25, 10) and (25, 20). Restrict your Navier solution to a three-term solution ( $n=1$  and  $m=1, 2, 3$ ).



<Navier Solution>

**Given Values :**

Young's modulus :	$E := 3 \cdot 10^7$ psi	Length : x distance	a := 50 in
		y distance	b := 40 in
Poisson's ratio :	$\nu := 0.3$	Plate Thickness :	h := 0.2 in
Plate bending stiffness :	$D := \frac{E \cdot h^3}{12(1 - \nu^2)}$	Load :	$p(x,y) := -100y + 2.5y^2$
	$D = 2.198 \times 10^4$		

Indecies :

m := 1..3 n := 1

Maximum values for the indecies :

mn := 1 mm := 3

**Load coefficient,  $q_{mn}$  :**

$$q_{m,n} := \frac{4}{a \cdot b} \int_0^b \int_0^a p(x,y) \cdot \sin\left(\frac{m \cdot \pi \cdot x}{a}\right) \cdot \sin\left(\frac{n \cdot \pi \cdot y}{b}\right) dx dy \quad q = \begin{pmatrix} -1314.04573 \\ -5.31026 \times 10^{-14} \\ -438.01524 \end{pmatrix}$$

**Coefficient,  $W_{mn}$  :**

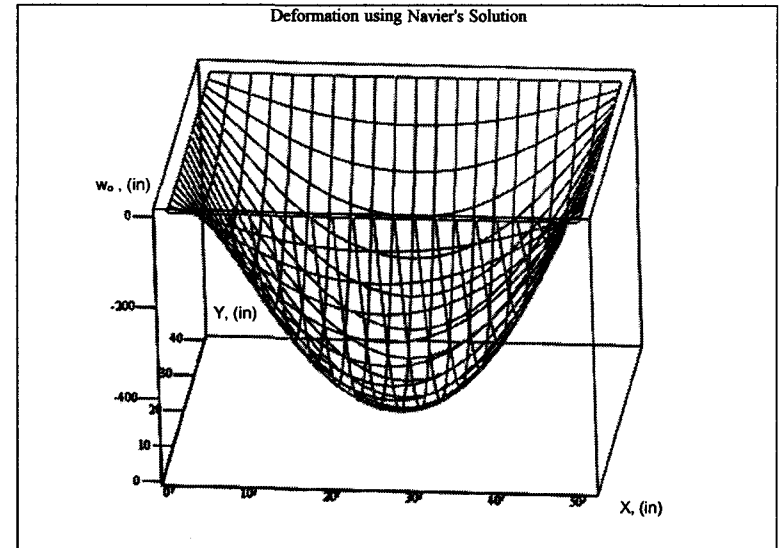
$$w_{m,n} := \frac{q_{m,n}}{D \cdot \pi^4 \left[ \left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2 \right]^2} \quad w_{m,n} = \begin{pmatrix} -584.21762 \\ -5.01034 \times 10^{-15} \\ -11.46166 \end{pmatrix}$$

Deformation function is defined as :

$$w_0(x,y) := \sum_{n=1}^{nn} \sum_{m=1}^{mn} w_{m,n} \cdot \sin\left(\frac{m \cdot \pi \cdot x}{a}\right) \cdot \sin\left(\frac{n \cdot \pi \cdot y}{b}\right)$$

Deformation at given points :

$x = 25, y = 10 :$	$w_0(25,10) = -404.99962$ in
$x = 25, y = 20 :$	$w_0(25,20) = -572.75596$ in



<Number of the element>   <The Length of the Domain>   <The length of the each element>

$$ne := 3 \qquad L := 1 \qquad he := \frac{L}{ne}$$

<Number of Node> <Number of EBC> <Node number of the EBC> <EBC and NBC >

$$nd := 2 \qquad nb := 1 \qquad z := (1 \ 0 \ 0)^T \qquad U_1 := 0 \qquad Q_{ne+1} := 1$$

<Lagrange Interpolation functions for the linear element>

$$\psi(x) := \left( 1 - \frac{x}{he} \quad \frac{x}{he} \right)^T$$

<Define the Global Stiffness Matrix and the Global Source Vector>

$$K_{ne+1, ne+1} := 0 \qquad F_{ne+1} := 0$$

<Determine components of the element stiffness matrix and local source vector for a two-noded element. >

$$m := 1..ne \quad n := 1..ne+1 \quad i := 1..nd \quad j := 1..nd$$

$$k_{i,j} := \int_0^{he} \left( \frac{d}{dx} \psi(x)_i \frac{d}{dx} \psi(x)_j - \psi(x)_i \psi(x)_j \right) dx \qquad k = \begin{pmatrix} 2.88889 & -3.05556 \\ -3.05556 & 2.88889 \end{pmatrix} \quad \begin{array}{l} \text{<Element} \\ \text{Stiffness} \\ \text{Matrix>} \end{array}$$

$$f := \begin{array}{l} \text{for } m \in 1..ne \\ \quad \text{for } i \in 1..nd \\ \quad \quad r_i \leftarrow \int_0^{he} [(m-1) \cdot he + x]^2 \cdot \psi(x)_i dx \\ \quad \quad s_m \leftarrow r \\ \quad f \leftarrow s \end{array} \qquad f = \begin{pmatrix} \{2,1\} \\ \{2,1\} \\ \{2,1\} \end{pmatrix} \qquad f_1 = \begin{pmatrix} -0.00309 \\ -0.00926 \\ -0.03395 \end{pmatrix} \quad \begin{array}{l} \text{<Local} \\ \text{Source} \\ \text{Vector>} \\ f_2 = \begin{pmatrix} -0.05247 \\ -0.10185 \\ -0.13272 \end{pmatrix} \end{array}$$

<Assembling Global Stiffness Matrix and Global Source Vector>

<Connectivity Matrix>

$$B := \begin{array}{l} \text{for } i \in 1..ne \\ \quad \text{for } j \in 1..nd \\ \quad \quad b_{i,j} \leftarrow i + j - 1 \end{array} \qquad B = \begin{pmatrix} 1 & 2 \\ 2 & 3 \\ 3 & 4 \end{pmatrix} \qquad B \leftarrow b$$

<Global Stiffness Matrix>

$$K_{B(m,i), B(m,i)} := K_{B(m,i), B(m,i)} + k_{i,j} \qquad K = \begin{pmatrix} 2.88889 & -3.05556 & 0 & 0 \\ -3.05556 & 5.77778 & -3.05556 & 0 \\ 0 & -3.05556 & 5.77778 & -3.05556 \\ 0 & 0 & -3.05556 & 2.88889 \end{pmatrix}$$

<Global Source Vector>

$$F_{(B_m,i)} := F_{(B_m,i)} + (f_m)_i \qquad F^T = (-0.00309 \quad -0.04321 \quad -0.15432 \quad -0.13272)$$

<Incorporate EBC into systems of Equations>

$$K_{mod} := K \qquad F_{mod} := F + Q$$

$$g := 1..nb \qquad F_{mod_n} := F_{mod_n} - K_{n,z_g} \cdot U(z_g) \qquad F_{mod_{z_g}} := U(z_g)$$

$$K_{mod_{z_g, n}} := 0 \qquad K_{mod_{n, z_g}} := 0 \qquad K_{mod_{z_g, z_g}} := 1$$

<Solve for the unknown global displacements>

$$U := K_{mod}^{-1} \cdot F_{mod} \qquad U^T = (0 \ 0.4134 \ 0.79584 \ 1.14197)$$

<Solve for the unknown SV DOF>

$$Q := K \cdot U - F \qquad Q^T = (-1.26007 \ 0 \ 0 \ 1)$$

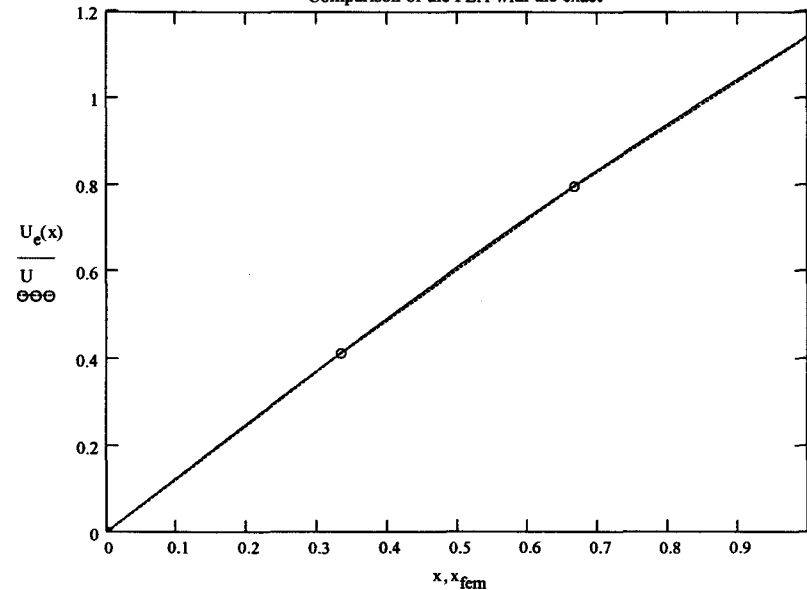
<Computing the location in the X-coordinate system and exact solution>

$$x_{fem} := \begin{array}{l} \text{for } i \in 1..ne+1 \\ \quad y_i \leftarrow \frac{L}{ne} \cdot (i-1) \\ \quad x_{fem} \leftarrow y \end{array} \qquad U_e(x) := \frac{2 \cdot \cos(1-x) - \sin(x)}{\cos(1)} + x^2 - 2$$

<Comparing the solution associated with FEM and exact solution for the location of the local nod point>

$$x_{fem} = \begin{pmatrix} 0 \\ 0.33333 \\ 0.66667 \\ 1 \end{pmatrix} \qquad U = \begin{pmatrix} 0 \\ 0.4134 \\ 0.79584 \\ 1.14197 \end{pmatrix} \qquad U_e(x_{fem}) = \begin{pmatrix} 0 \\ 0.4146 \\ 0.79784 \\ 1.14422 \end{pmatrix}$$

Comparison of the FEA with the exact



## Performing integration of stress resultant data

Skin thickness :  $t := 0.064$  (in)

### Read in the matrix of X-Nvv data to be interpolated:

$NvvL_{BC} := \text{READPRN}("/\text{txt}/\text{B-NO-Nvv-L1.txt}")$  **note** : Just like "HTML", simply specify the location of your data file

### Sorting the input data for each model by increasing order :

$NvvL_{BC} := \text{csort}(NvvL_{BC}, 0)$

### Take the values for each column :

$XL_B := NvvL_{BC}^{\langle 0 \rangle}$        $NvvL_B := NvvL_{BC}^{\langle 1 \rangle}$

### Spline coefficients:

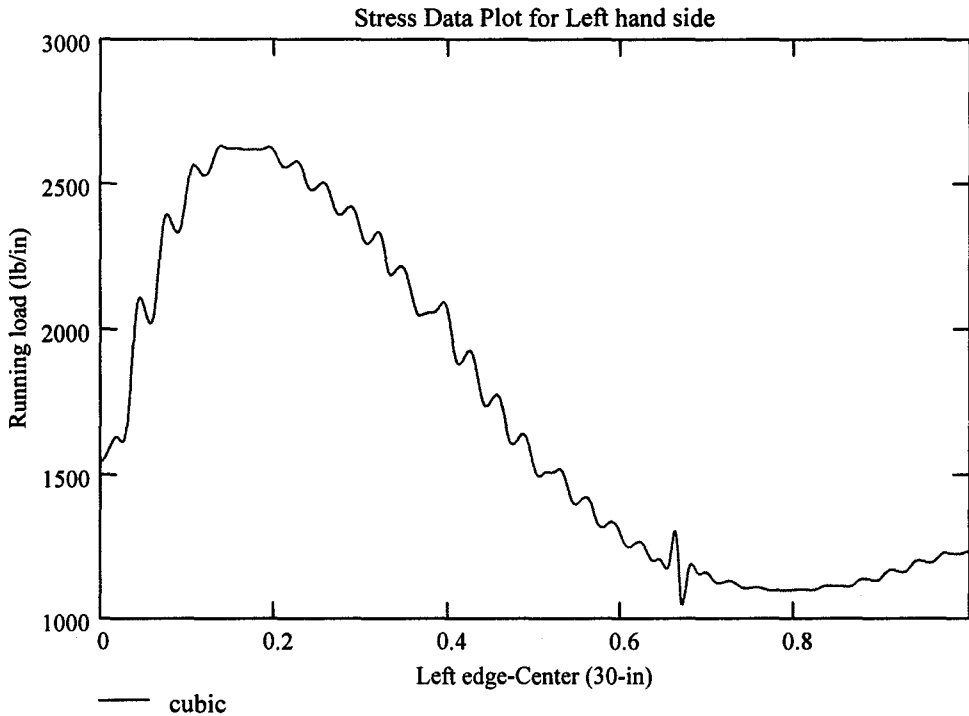
$SL_l := \text{lspline}(XL_B, NvvL_B)$  : The resultant spline curve is **linear** at the endpoints.

$SL_p := \text{pspline}(XL_B, NvvL_B)$  : The resultant spline curve is **parabolic** at the endpoints.

$SL_c := \text{cspline}(XL_B, NvvL_B)$  : The resultant spline curve is **cubic** at the endpoints.

### Fitting functions :

$\text{fitL}_c(xL_B) := \text{interp}(SL_c, XL_B, NvvL_B, xL_B)$  : ( spline, x-value, y-value, parameter )



### Integral fitting function:

$$LBfit(x) := \int_0^x \text{fitL}_c(\xi) d\xi$$

### Finding stress :

$$P_B := \frac{LBfit(1)}{t}$$

$$P_B = 26629.651$$